RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2019
THIRD YEAR [BATCH 2017-20]

Date : 12/12/2019 **PHYSICS (Honours)**

Time : 11 am – 1 pm **Paper : V [Gr. B]** Full Marks : 50

Answer **any five** questions of the following: [5×10]

1. a) Explain the advantages of the Lagrangian approach over the Newtonian formalism in classical mechanics. (2)

   b) Explain if the following constraints are holonormic

   i) A rigid body

   ii) The motion of a body subjected to the constraint given by $x\dot{x}+y\dot{y}+z\dot{z}=f(t)$, the symbols having their usual meanings

   iii) A particle moving within an enclosed space. (3)

   c) State and explain D'Alembert's principle for a dynamical system of N mass points, subjected to holonomic constraints. (2)

   d) Write down the general form of Lagrange's equation. Deduce the form it takes for conservative force acting on the system. (3)

2. a) Two particles of masses m_1 and m_2 and position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively interact via a potential $V(\vec{r}_1-\vec{r}_2)$. Write down the Lagrangian in terms of the centre of mass coordinate $\vec{R}$ and the relative coordinate $\vec{r}=\vec{r}_1-\vec{r}_2$. Deduce the constant of motion, if any, from the form of the Lagrangian. (3)

   b) Suppose you are given a Lagrangian $L=\frac{1}{2}m\dot{x}^2-V(x)$. You change it to another Lagrangian $L'=\frac{1}{2}m\dot{x}^2-V(x)+3x^2+4t^2$. Do the two Lagragians lead to the same equation? Explain (without going into the derivation) (2+2)

   c) Given that the path $y=y(x)$ along which the integral $I=\int F\left(y,\frac{dy}{dx},x\right)dx$ with the fixed end points is stationary satisfies the solution of the Euler-Lagrange equation $\frac{d}{dx}\left(\frac{\partial F}{\partial\left(\frac{dy}{dx}\right)}\right)-\frac{\partial F}{\partial y}=0$, find out the path joining two fixed points on a plane, that has the shortest length. (3)

3. a) Starting from the Hamiltonian $H=\sum_i p_i\dot{q}_i-L$, show under what conditions does H equal the total mechanical energy of the system. (2)

   b) Starting from Hamilton's equations of motion, show that if i) a coordinate be cyclic with respect to the Lagrangian, it also remains so in respect of the Hamiltonian. ii) The Lagrangian $L=\frac{1}{2}\dot{x}^2+\alpha x^4$, α being a constant, H is constant. (3)

c) A system with one degree of freedom has the Hamiltonian $H(q,p)=\frac{p^2}{2m}+A(q)p+B(q)$, where A and B are given functions of q and p, the symbols having their usual meaning. Find out i) $\dot{q}$ ii) $L(q,\dot{q})$. (5)

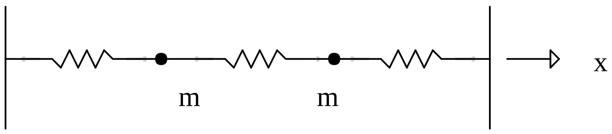
4. a) The Hamiltonian for a simple harmonic oscillator is $H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2$.

Let, $a=\sqrt{\frac{m\omega}{2}}\left(x+\frac{ip}{m\omega}\right)$ and $a^*=\sqrt{\frac{m\omega}{2}}\left(x-\frac{ip}{m\omega}\right)$,

i) Show that $H=\omega a^* a$,

ii) Calculate the Poisson brackets $\left[a,a^*\right]$, $[a,H]$, $\left[a^*,H\right]$.

b) Consider two identical mass-spring system coupled by a spring of stiffness constant K and arranged linearly along the x-axis, as shown in the figure.



i) Write down the Lagrangian for small longitudinal oscillations of the system.

ii) Obtain the normal frequencies of oscillations and discuss the nature of the corresponding modes of oscillation. (5+5)

5. a) State D'Moivre's theorem. Find out all the roots of the equation cosh z = 4. (1+2)

b) If we choose the principal branch of $\sinh^{-1} z$ to be that one for which $\sinh^{-1}(0)=0$, prove that $\sinh^{-1} z=\ln\left(z+\sqrt{z^2-1}\right)$. (3)

c) Deduce the branch points in the finite plane of

i) $\sqrt{z-a}$

ii) $\ln z$

(a is a constant). How many branches do the functions have? (4)

6. a) Does the function $e^{-\frac{1}{z^2}}$ posses a unique limit as $z\to 0$? Explain. (1)

b) Prove that the necessary and sufficient conditions for differentiability of a function $f(z)=u(x,y)+iv(x,y)$ at a point in a region R are, $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$.

Show that for the above conditions u and v are harmonic functions. Can you give an example of a function which is harmonic but does not satisfy Canchy-Riemann equations. (3+1+1)

c) Prove that the function $u=2x(1+y)$ is harmonic. Find the function v for which f(z) is analytic. Write f(z) in terms of z. (4)

7. a) Find out the residues of the functions at the singularities, stating their nature, of

i) $e^{-\frac{1}{z}}$ ii) $\frac{1}{z^2+1}$ (4)

b) Expand the functions $\frac{1}{z(z-1)}$ about z = 0 and z = 2, stating the regions of validity of the expansions. (3)

c) State and prove Cauchy's theorem for a simply connected domain. (1+2)

8. a) State and prove the residue theorem. (1+2)

b) Evaluate $\frac{1}{2\pi i}\oint_{c}\frac{\cos \pi z}{z^2-1}dz$ around a rectangle with vertices at

i) $2 \pm i, -2 \pm i$

ii) $-i, 2-i, 2+i, i$ (2+2)

c) Prove that $\oint_{C}\frac{\cos \pi z}{z^2}dz = \pi i$, if C is the square with the vertices at $\pm 2 \pm 2i$. (3)

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